SUSY Flat Direction Decay - the prospect of particle production and preheating investigated in the unitary gauge

Anders Basbøll[1,*]

[1]*Department of Physics and Astronomy, University of Aarhus, Ny Munkegade, DK-8000 Aarhus C*

(Dated: 8th May)

We look at the possibility of non-perturbative particle production after inflation from SUSY flat directions produced by rotating eigenstates thereby avoiding the standard adiabaticity conditions. This might lead to preheating and prevent the delay of thermalisation of the universe. We investigate the flat directions LLE^c and $U^cD^cD^c$ and find no particle production. These 2 directions are very important, since they have been named as possible candidates for being the inflaton. We investigate $QLQLQLE^c$ and find particle production and therefore the possibility of preheating. We investigate the LLE^c and $U^cD^cD^c$ directions appearing simultaneously, and find no production. Finally, we investigate LLE^c and QLD^c simultaneously - with one L-field in common. Here we do find particle production and therefore the possibility of preheating. This means that if SUSY flat directions are to delay thermalisation and thus explain the (lack of) gravitino production, it is necessary to explain why complicated directions as $QLQLQLE^c$ are not exited, and why combinations like LLE^c and QLD^c are not both exited.

PACS numbers:
Keywords: Flat directions, Preheating, Supersymmetry

*Electronic address: andersb@phys.au.dk

I. INTRODUCTION

The scalar potential of the Minimal Supersymmetric Standard Model (MSSM) possesses a large number of F- and D-flat directions [1, 2]. These flat directions might generate the baryon asymmetry of the Universe through the out-of-equilibrium CP violating decay of coherent field oscillations along the flat directions themselves [3–5].

Recently the cosmological importance of flat direction vacuum expectation values (VEV)s [Often we will use VEV meaning *nonzero* vacuum expectation value - this should not cause confusion] and the decay thereof has been investigated. In [6] it was asserted that large flat direction VEV's can persist long enough to delay thermalization after inflation and therefore lead to low reheat temperatures. This is of great importance. A lower reheating temperature would potentially solve the (lack of) gravitino problem[7]. It has also been claimed [8] that large flat direction VEV's can prevent non-perturbative parametric resonant decay (preheating) of the inflaton since the inflaton decay products become sufficiently massive preventing preheating from ever becoming efficient. These arguments hold so long as the flat direction VEV's do not rapidly decay - they must persist long enough so that they can delay thermalization and block inflaton preheating. In [9] it was claimed that non-perturbative decay can lead to a rapid depletion of the flat direction condensate and thus precludes the delay of thermalization after inflation. It was also concluded that in order for the flat direction to decay non-perturbatively the system requires more than one flat direction [9, 10]. In [10] it was stated that even in the presence of multiple flat directions, some degree of fine-tuning was necessary to achieve flat direction decay. We note that very recently [11] it has been claimed that even if non-perturbative particle production happen, the main decay mode will still be perturbative.

The presence of Nambu-Goldstone bosons (Goldstones) is a very important point in this discussion. The flat directions are charged under the gauge group of the MSSM. Therefore the flat direction VEV will break some or all of the gauge symmetries of the theory and therefore the presence of the associated Goldstones must be expected. [9] considers a gauged $U(1)$ model and constructs the mixing matrix for the excitations around the flat direction VEV. In [9] it was claimed that in the single flat direction case, non-perturbative decay proceeds solely via a massless Goldstone mode as only the Goldstone mode mixes with the Higgs and all other massless moduli remain decoupled. Since the Goldstone represents an unphysical gauge degree of freedom, it was concluded [9, 10] that no preheating occurs in the single flat direction case - a Goldstone can be gauged away. In order to determine if flat direction VEV's decay non-perturbatively into scalar degrees of freedom, one must remove the Goldstones ie. use the unitary gauge.

In an earlier paper [12] we and our colleagues considered toy models to demonstrate that, in the unitary gauge, the mixing matrix of the excitations around a flat direction VEV permits preheating. Moreover, we found that flat direction decay depends on the number of dynamical, physical phases appearing in the flat direction VEV. Specifically, a physical phase difference between two of the individual field VEV's making up the flat direction is needed.

In the present paper, we look at (some of) the actual SUSY flat directions. In section II we look at the LLE^c direction, in section III we review the particle production mechanism from rotating eigenvectors, in section IV we conclude on the LLE^c case, and in section V we do the $U^cD^cD^c$. The mentioned directions are especially interesting, since they are mentioned as especially well suited inflaton candidates in [6]. We investigate $QLQLQLE^c$ in section VI. We conclude on one flat direction in section VII. Then we proceed to 2 directions, first the non-overlapping $LLE^c + U^cD^cD^c$ of the 2 inflaton candidates of [6] in section VIII and then the overlapping directions $QLD^c + LLE^c$ in section X. Finally we look at the simpler approach of just counting the fields without any calculations in section XI and conclude in section XII.

II. LLE^c

One flat direction often mentioned in the literature is LLE^c. Flatness demands the fields with VEV's to come from different generations, and the 2 L fields with VEV to have opposite SU(2)-charge. Also, the 3 VEV's must have the same absolute value. This leaves essentially only 1 choice (when masses are ignored).

We give these VEV's:

$$\begin{aligned} <\nu_e> &= \varphi e^{i\sigma_1} \\ <\mu> &= \varphi e^{i\sigma_2} \\ <\tau^c> &= \varphi e^{i\sigma_3}. \end{aligned} \tag{1}$$

Also, it is clear that 2 other fields will play a role.

$$\begin{aligned} <e> &= 0 \\ <\nu_\mu> &= 0. \end{aligned} \tag{2}$$

The Lagrangian reads

$$\mathcal{L} = \sum_{i=1}^{3} \frac{1}{2}|D_\mu \Phi_i|^2 - V - \frac{1}{4}F_{\mu\nu}^2 - \sum_i \frac{1}{4}W_{\mu\nu}^{i2} \tag{3}$$

where for field ϕ_i $D_i^\mu = \left[(\partial^\mu - iq_i A_0^\mu)\delta_{ij} - \sum_{a=1}^3 iP_{ij}^a A_a^\mu\right]\phi_j$ denotes the covariant derivative. P^a is the a^{th} Pauli-matrix. The potential we consider arises from the supersymmetric D-terms and has the form

$$V = \frac{1}{2}\left(D_H^2 + \sum_a D_a^2\right) \tag{4}$$

where

$$D_H = \frac{g_1}{2}\sum_i q_i|\phi_i|^2 \tag{5}$$

$$D_a = \frac{g_2}{2}\phi^\dagger P^a \phi \tag{6}$$

where q_i is the hypercharge, and g_1, g_2 are the hypercharge and SU2 gauge couplings.

The essential part is removing the Goldstones correctly. To do that we start by looking at the fields with the VEV's only (no excitations). We've written those earlier, and we get mixed kinetic terms

$$\mathcal{L} \supset -\varphi^2 A_0(\dot{\sigma_1} + \dot{\sigma_2} - 2\dot{\sigma_3}) - \varphi^2 A_3(\dot{\sigma_1} - \dot{\sigma_2}) \tag{7}$$

which has the form of a coupling between the gauge field and the background condensate. Terms of this type will feed into the equations of motion for the gauge field which, in turn, will have an effect on the equations of motion for the scalar excitations. The remaining terms in $\mathcal{L}$ are

$$\frac{1}{2}\varphi^2\left[6A_0^2 + 2A_1^2 + 2A_2^2 + 2A_3^2 + \dot{\sigma_1}^2 + \dot{\sigma_2}^2 + \dot{\sigma_3}^2\right] \tag{8}$$

- all desired terms. By making a $U(1)$ gauge transformation on the VEV,

$$\langle\Phi_i\rangle \to \langle\Phi_i'\rangle = e^{iq_i\lambda}\langle\Phi_i\rangle \tag{9}$$

with

$$\lambda = \frac{2\sigma_3 - \sigma_1 - \sigma_2}{3}, \tag{10}$$

and by making a $SU(2)$ gauge transformation on the VEV,

$$\langle\Phi_i\rangle \to \langle\Phi_i'\rangle = e^{iP^3\gamma}\langle\Phi_i\rangle \tag{11}$$

with

$$\gamma = \frac{\sigma_2 - \sigma_1}{2}, \tag{12}$$

we can gauge the unwanted terms away and avoid a complicated analysis of the kinetic terms. The resulting form of the VEV reads,

$$\begin{aligned} \langle\nu_e\rangle &= \varphi e^{i\sigma} \\ \langle\mu\rangle &= \varphi e^{i\sigma} \\ \langle\tau^c\rangle &= \varphi e^{i\sigma} \end{aligned} \tag{13}$$

where $\sigma = (\sigma_1 + \sigma_2 + \sigma_3)/3$ represents the remaining independent physical phase. Following [13], we can write the fields in the unitary gauge as (including the other relevant fields),

$$\begin{aligned} \nu_e &= (\varphi + \xi_2)e^{i(\sigma + \frac{\xi_1}{\sqrt{3}\varphi})} \\ e &= (\xi_5 + i\xi_6)e^{i\sigma} \\ \nu_\mu &= (\xi_7 + i\xi_8)e^{i\sigma} \\ \mu &= (\varphi + \xi_3)e^{i(\sigma + \frac{\xi_1}{\sqrt{3}\varphi})} \\ \tau^c &= (\varphi + \xi_4)e^{i(\sigma + \frac{\xi_1}{\sqrt{3}\varphi})} \end{aligned} \tag{14}$$

where σ represents time dependent phase of the VEV (we have just showed the phase differences are gauged away), ξ_1 parameterises its excitation, $\xi_{2,3,4}$ parameterise the excitations around the VEV, and $\xi_{5,6,7,8}$ parameterise the 2 no-VEV fields (the phase on these fields is not necessary, but allowed, and will be convenient).

Again we will look at the kinetic term. First, the φ^2-term

$$\frac{1}{2}\varphi^2\left[6A_0^2+2A_1^2+2A_2^2+2A_3^2+3\dot{\sigma}^2\right] \tag{15}$$

- not surprisingly. This contains no goldstones, so we proceed to next order.

The terms indicating Goldstones should include $\dot{\xi}_i$. These terms are

$$\mathcal{L}\supset-\varphi\left(A_1(\dot{\xi}_6+\dot{\xi}_8)+A_2(\dot{\xi}_7-\dot{\xi}_5)\right). \tag{16}$$

The remaining terms are on the forms

$$\mathcal{L}\supset\varphi\left(C_{ijk}\xi_iA_jA_k+D_{ij}\xi_iA_j\dot{\sigma}+E_i\xi_i\dot{\sigma}^2-\sqrt{3}\dot{\xi}_1.\dot{\sigma}\right) \tag{17}$$

Here it is clear, that the field excitation terms (excluding derivative terms) are suppressed compared to φ^2-terms in the potential. The only excitation is the last term. However, it is just a "mixing" between an excitation and its own VEV.

The Goldstones are removed by demanding $\dot{\xi}_6=-\dot{\xi}_8$ and $\dot{\xi}_7=\dot{\xi}_5$. Doing this, and renormalising, we take

$$\begin{aligned}
\nu_e &= (\varphi+\xi_2)e^{i(\sigma+\frac{\xi_1}{\sqrt{3}\varphi})}\\
e &= \frac{(\xi_5+i\xi_6)}{\sqrt{2}}e^{i\sigma}\\
\nu_\mu &= \frac{(\xi_5-i\xi_6)}{\sqrt{2}}e^{i\sigma}\\
\mu &= (\varphi+\xi_3)e^{i(\sigma+\frac{\xi_1}{\sqrt{3}\varphi})}\\
\tau^c &= (\varphi+\xi_4)e^{i(\sigma+\frac{\xi_1}{\sqrt{3}\varphi})}.
\end{aligned} \tag{18}$$

This does indeed kill the mixed derivative terms. The remaining terms stay as they are. But they are all VEV-suppressed, so it is justified to move to the coordinate derivative, rather than the covariant derivative.

The remaining kinetic term (to zero'th order in φ) are

$$\begin{aligned}
\mathcal{L}\supset\sum_{i=1}^{6}\left(\frac{1}{2}\dot{\xi}_i^{\,2}\right)+\sum_{i=2}^{6}\left(\frac{1}{2}\xi_i^2\dot{\sigma}^2\right)\\
+\sum_{i=2}^{4}\left(\varphi\xi_i\dot{\sigma}^2+\frac{2}{\sqrt{3}}\xi_i\dot{\xi}_1\right)+\frac{3}{2}\varphi^2\dot{\sigma}^2+\sqrt{3}\varphi\dot{\sigma}\dot{\xi}_1.
\end{aligned} \tag{19}$$

The first term are the kinetic terms that show we have correctly normalised kinetic fields - including that there are no cross terms. The second term is completely negligible compared to the φ^2-terms of V. The third term, though bigger than the prior one, is still suppressed. The fourth term is a rotation between the excitation states. These are very important and will give the U-matrix below. The fifth term is just a VEV-term, and the final term is the "mixing" between an excitation and its own VEV. So everything is fine.

On substituting the fields of eq.18 into the Lagrangian given in eq.3 and defining the vector $\Xi\equiv(\xi_1,\xi_2,\xi_3,\xi_4,\xi_5,\xi_6)^T$, we find the quadratic terms

$$\mathcal{L}\supset\frac{1}{2}|\partial_\mu\Xi|^2-\frac{1}{2}\Xi^T\mathcal{M}^2\Xi-\dot{\Xi}^TU\Xi+... \tag{20}$$

where the ellipses denote higher order terms and interactions. The matrix U given in the second part of the third term in eq.19 reads

$$U_{init}=\begin{pmatrix}
0 & -\frac{2\dot{\sigma}}{\sqrt{3}} & -\frac{2\dot{\sigma}}{\sqrt{3}} & -\frac{2\dot{\sigma}}{\sqrt{3}} & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0.
\end{pmatrix} \tag{21}$$

However, we want an antisymmetric matrix for the procedure below. Using partial integration - and ignoring surface

terms - we find

$$U = \begin{pmatrix} 0 & -\frac{\dot{\sigma}}{\sqrt{3}} & -\frac{\dot{\sigma}}{\sqrt{3}} & -\frac{\dot{\sigma}}{\sqrt{3}} & 0 & 0 \\ \frac{\dot{\sigma}}{\sqrt{3}} & 0 & 0 & 0 & 0 & 0 \\ \frac{\dot{\sigma}}{\sqrt{3}} & 0 & 0 & 0 & 0 & 0 \\ \frac{\dot{\sigma}}{\sqrt{3}} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \tag{22}$$

while the mass matrix for the physical excitations appears as

$$\mathcal{M}^2 = \varphi^2 \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & g_1^2 + g_2^2 & g_1^2 - g_2^2 & -2g_1^2 & 0 & 0 \\ 0 & g_1^2 - g_2^2 & g_1^2 + g_2^2 & -2g_1^2 & 0 & 0 \\ 0 & -2g_1^2 & -2g_1^2 & 4g_1^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2g_2^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2g_2^2 \end{pmatrix} = B\mathcal{M}_d^2 B^T, \tag{23}$$

with eigenvalues $M_1^2 = 6g_1^2\varphi^2$, $M_2^2 = M_3^2 = M_4^2 = 2g_2^2\varphi^2$,$M_5^2 = M_6^2 = 0$ (the entries of the diagonal matrix $\mathcal{M}_d$). B is an orthogonal matrix which diagonalises $\mathcal{M}^2$ and M_{1-4}^2 corresponds to the mass of the physical eigenstates associated with the spontaneous breaking of the symmetries. $M_5^2 = M_6^2 = 0$ correspond to the massless excitations around the flat direction VEV.

The last term in eq.20 appears as a consequence of the time-dependence of the background – it represents a mixing between the fields $\xi_{1,2,3,4,5,6}$, and their time-derivatives. The effect of these terms on the system becomes clear if we make field redefinitions that remove the mixed derivative terms. The resulting transformation leaves the system in an inertial frame in field space and leads to a time-dependent mass matrix. Defining $\Xi' = A\Xi$ (A is orthogonal), we find the condition that A must satisfy in order for all the mixed derivative terms to cancel

$$\dot{A}^T A = U. \tag{24}$$

The Lagrangian for the Ξ' system now reads

$$\mathcal{L} \supset \frac{1}{2}|\partial_\mu \Xi'|^2 - \frac{1}{2}\Xi'^T \mathcal{M}'^2 \Xi' \tag{25}$$

where $\mathcal{M}'^2 = A\mathcal{M}^2 A^T = AB\mathcal{M}_d^2 B^T A^T = C\mathcal{M}_d^2 C^T$, and $C = AB$. The matrix C is an orthogonal time-dependent matrix, with columns corresponding to the eigenvectors of $\mathcal{M}'^2$. We now have a system of scalar fields with canonically normalized kinetic terms and time dependent eigenvectors.

The central point of this discussion centers precisely on the appearance of the *time dependent* eigenvectors for the six scalar fields. This satisfies a necessary but not sufficient condition for preheating. In the next section, we briefly run through the details of the non-perturbative production of the light scalar fields following the analysis of [14]. This is a more brief summary than in [12] - which is otherwise followed here.

III. NON-PERTURBATIVE PRODUCTION OF PARTICLES

Including gravity, the dynamics of the re-scaled conformally coupled scalar fields, $\chi_i = a\Xi'_i$, where a denotes the scale factor and Ξ'_i the i-th component of the vector Ξ', are governed by the following equations of motion (sum over repeated indices is implied),

$$\ddot{\chi}_i + \Omega_{ij}^2(t)\chi_j = 0 \tag{26}$$

where dots represent derivatives with respect to conformal time t, and

$$\Omega_{ij}^2 = a^2 \mathcal{M}'^2_{ij} + k^2\delta_{ij}, \tag{27}$$

where k labels the comoving momentum. Using an orthogonal time-dependent matrix $C(t)$, we can diagonalise Ω_{ij} via $C^T(t)\Omega^2(t)C(t) = \omega^2(t)$, giving the diagonal entries $\omega_j^2(t)$. Terms of the form $\sim \varphi\dot{\sigma}\dot{\chi}$ arising from the kinetic terms do not affect the evolution of the nonzero k quantum modes [15].

As the vacuum changes, a new set of creation/annihilation operators are required. We use Bogolyubov transformation with Bogolyubov coefficients α and β (which denote matrices in the multi-field case).

Initially $\alpha = \mathbb{I}$ and $\beta = 0$ while the coupled differential equations (matrix multiplication implied):

$$\begin{aligned} \dot{\alpha} &= -i\omega\alpha + \frac{\dot{\omega}}{2\omega}\beta - I\alpha - J\beta \\ \dot{\beta} &= \frac{\dot{\omega}}{2\omega}\alpha + i\omega\beta - J\alpha - I\beta, \end{aligned} \tag{28}$$

govern the system's time evolution with the matrices I and J given by

$$I = \frac{1}{2}\left(\sqrt{\omega}\, C^T \dot{C} \frac{1}{\sqrt{\omega}} + \frac{1}{\sqrt{\omega}}\, C^T \dot{C} \sqrt{\omega}\right) \tag{29}$$

$$J = \frac{1}{2}\left(\sqrt{\omega}\, C^T \dot{C} \frac{1}{\sqrt{\omega}} - \frac{1}{\sqrt{\omega}}\, C^T \dot{C} \sqrt{\omega}\right). \tag{30}$$

Similarly to the single-field case it can be shown [14] that at any generic time the occupation number of the ith bosonic eigenstate reads (no summation implied)

$$n_i(t) = (\beta^* \beta^T)_{ii}. \tag{31}$$

As pointed out in [9, 14], there exists two sources of non-adiabaticity in the multi-field scenario. The first source arises from the individual frequency time dependence and appears as the only source of non-adiabaticity in the single field case. The second source appears from the time dependence of the frequency matrix Ω_{ij} giving rise to terms in eq.28 proportional to I and J. This second source provides the most important contribution in our analysis and gives rise to non-perturbative particle production.

Since initially $\alpha = \mathbb{I}$ and $\beta = 0$, eq.28 shows that a non-vanishing matrix J is a necessary condition to obtain $\dot{\beta} \neq 0$ and hence $n_i(t) \neq 0$. In general, we have

$$C^T \dot{C} = B^T A^T \dot{A} B = -B^T U B \tag{32}$$

where A, B and U were defined in the previous section. The last equation only holds if B is constant in time. This is obviously the case in the LLE^c-case, since, B diagonalises a constant matrix.

IV. LLE^c CONCLUSION

For the LLE^c example outlined above, J is a 6×6 zero matrix. Therefore there is no particle production and no preheating.

V. $U^c D^c D^c$

One would expect the $U^c D^c D^c$ case to be much the same - as indeed we shall see it is. We give VEV's to these fields (again from different generations to avoid F-terms)

$$\begin{aligned} &< u^{c\overline{1}} >= \varphi e^{i\sigma_1} \\ &< s^{c\overline{2}} >= \varphi e^{i\sigma_2} \\ &< b^{c\overline{3}} >= \varphi e^{i\sigma_3}. \end{aligned} \tag{33}$$

The Lagrangian reads

$$\mathcal{L} = \sum_{i=1}^{3} \frac{1}{2}|D_\mu \Phi_i|^2 - V - \sum_i \frac{1}{4} F_{\mu\nu}^{i2} - \sum_i \frac{1}{4} G_{\mu\nu}^{i2} \tag{34}$$

where for field ϕ_i $D_i^\mu = \left[(\partial^\mu - iq_i A_0^\mu)\delta_{ij} - \sum_{A=1}^{8} iGM_{ij}^A B_A^\mu\right] \phi_j$ denotes the covariant derivative. where GM^A is the A^{th} Gell-Mann-matrix. The potential now looks like

$$V = \frac{1}{2}\left(D_H^2 + \sum_A D_A^2\right) \tag{35}$$

where

$$D_A = \frac{g_3}{2}\phi^\dagger G M^A \phi \tag{36}$$

where g_3 is SU(3) gauge couplings. Removing mixed kinetic terms as before (in 2 tempi), we use

$$\begin{aligned}
u^{c\bar{1}} &= (\varphi+\xi_2)e^{i(\sigma+\frac{\xi_1}{\sqrt{3}\varphi})}, \\
u^{c\bar{2}} &= \frac{(\xi_5+i\xi_6)}{\sqrt{2}}e^{i\sigma}, \\
u^{c\bar{3}} &= \frac{(\xi_7+i\xi_8)}{\sqrt{2}}e^{i\sigma}, \\
s^{c\bar{1}} &= \frac{(\xi_5-i\xi_6)}{\sqrt{2}}e^{i\sigma}, \\
s^{c\bar{2}} &= (\varphi+\xi_3)e^{i(\sigma+\frac{\xi_1}{\sqrt{3}\varphi})} \\
s^{c\bar{3}} &= \frac{(\xi_9+i\xi_{10})}{\sqrt{2}}e^{i\sigma}, \\
b^{c\bar{1}} &= \frac{(\xi_7-i\xi_8)}{\sqrt{2}}e^{i\sigma}, \\
b^{c\bar{2}} &= \frac{(\xi_9-i\xi_{10})}{\sqrt{2}}e^{i\sigma}, \\
b^{c\bar{3}} &= (\varphi+\xi_4)e^{i(\sigma+\frac{\xi_1}{\sqrt{3}\varphi})}.
\end{aligned} \tag{37}$$

The U-matrix is

$$U = \begin{pmatrix}
0 & -\frac{\dot\sigma}{\sqrt{3}} & -\frac{\dot\sigma}{\sqrt{3}} & -\frac{\dot\sigma}{\sqrt{3}} & 0&0&0&0&0&0\\
\frac{\dot\sigma}{\sqrt{3}} & 0&0&0&0&0&0&0&0&0\\
\frac{\dot\sigma}{\sqrt{3}} & 0&0&0&0&0&0&0&0&0\\
\frac{\dot\sigma}{\sqrt{3}} & 0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0
\end{pmatrix} \tag{38}$$

while the mass matrix for the physical excitations appears as

$$\mathcal{M}^2 = \varphi^2 \begin{pmatrix}
0&0&0&0&0&0&0&0&0&0\\
0 & \frac{16}{9}g_1^2+\frac{4}{3}g_3^2 & \frac{-8}{9}g_1^2+\frac{-2}{3}g_3^2 & -\frac{-8}{9}g_1^2+\frac{-2}{3}g_3^2 & 0 & 0 & & & & \\
0 & \frac{-8}{9}g_1^2+\frac{-2}{3}g_3^2 & \frac{4}{9}g_1^2+\frac{4}{3}g_3^2 & \frac{4}{9}g_1^2+\frac{-2}{3}g_3^2 & 0&0&0&0&0&0\\
0 & \frac{-8}{9}g_1^2+\frac{-2}{3}g_3^2 & \frac{4}{9}g_1^2+\frac{-2}{3}g_3^2 & \frac{4}{9}g_1^2+\frac{4}{3}g_3^2 & 0&0&0&0&0&0\\
0&0&0&0&2g_3^2&0&0&0&0&0\\
0&0&0&0&0&2g_3^2&0&0&0&0\\
0&0&0&0&0&0&2g_3^2&0&0&0\\
0&0&0&0&0&0&0&2g_3^2&0&0\\
0&0&0&0&0&0&0&0&2g_3^2&0\\
0&0&0&0&0&0&0&0&0&2g_3^2
\end{pmatrix} = B\mathcal{M}_d^2 B^T, \tag{39}$$

with eigenvalues $M_1^2 = \left(\frac{8}{3}g_1^2 + 2g_3^2\right)\varphi^2$, $M_2^2 = M_3^2 = M_4^2 = M_5^2 = M_6^2 = M_7^2 = M_8^2 = 2g_3^2\varphi^2$,$M_9^2 = M_{10}^2 = 0$. Also here, we end up with $J = 0$ and no particle production and therefore preheating.

VI. $QLQLQLE^c$

$QLQLQLE^c$ case has so many fields with nonzero VEV, that all the phase differences cannot be gauged away. The starting point could be (notice that here, there are 2 essentially different possibilities - the squarks having identical

$SU(2)$-charge - or not)

$$\begin{aligned} < u^{c1} >&= \varphi e^{i\sigma_1} \\ < c^{c2} >&= \varphi e^{i\sigma_2} \\ < t^{c3} >&= \varphi e^{i\sigma_3} \\ < e >&= \varphi e^{i\sigma_4} \\ < \mu >&= \varphi e^{i\sigma_5} \\ < \tau >&= \varphi e^{i\sigma_6} \\ < e^c >&= \varphi e^{i\sigma_7} \end{aligned} \tag{40}$$

The Lagrangian reads

$$\mathcal{L} = \sum_{i=1}^{3} \frac{1}{2} |D_\mu \Phi_i|^2 - V - \frac{1}{4} F_{\mu\nu}^2 - \frac{1}{4} \sum_i W_{\mu\nu}^{i2} - \frac{1}{4} \sum_i G_{\mu\nu}^{i2} \tag{41}$$

where for field ϕ_i $D_i^\mu = \left[(\partial^\mu - i q_i A_0^\mu) \delta_{ij} - \sum_{a=1}^{3} i P_{ij}^a A_a^\mu - \sum_{A=1}^{8} i G M_{ij}^A B_A^\mu \right] \phi_j$ denotes the covariant derivative. The potential now looks like

$$V = \frac{1}{2} \left(D_H^2 + \sum_a D_a^2 + \sum_A D_A^2 \right). \tag{42}$$

Removing mixed kinetic terms as before (in 2 tempi), we use

$$
\begin{aligned}
u^{c1} &= (\varphi+\xi_4)e^{i(\sigma_1+\frac{\xi_1}{\sqrt{7}\varphi})}\\
c^{c2} &= (\varphi+\xi_5)e^{i(\sigma_1+\frac{\xi_1}{\sqrt{7}\varphi})}\\
t^{c3} &= (\varphi+\xi_6)e^{i(\sigma_1+\frac{\xi_1}{\sqrt{7}\varphi})}\\
e &= (\varphi+\xi_7)e^{i(\sigma_1+\frac{\xi_1}{\sqrt{7}\varphi}+\sigma_2+\frac{\xi_2}{\sqrt{2}\varphi}+\sigma_3+\frac{\xi_3}{\sqrt{6}\varphi})}\\
\mu &= (\varphi+\xi_8)e^{i(\sigma_1+\frac{\xi_1}{\sqrt{7}\varphi}-\sigma_2-\frac{\xi_2}{\sqrt{2}\varphi}+\sigma_3+\frac{\xi_3}{\sqrt{6}\varphi})}\\
\tau &= (\varphi+\xi_9)e^{i(\sigma_1+\frac{\xi_1}{\sqrt{7}\varphi}-2\sigma_3-\frac{2\xi_3}{\sqrt{6}\varphi})}\\
e^c &= (\varphi+\xi_{10})e^{i(\sigma_1+\frac{\xi_1}{\sqrt{7}\varphi})}\\
u^{c2} &= \frac{\xi_{11}+i\xi_{12}}{\sqrt{2}}e^{i\sigma_1}\\
u^{c3} &= \frac{\xi_{13}+i\xi_{14}}{\sqrt{2}}e^{i\sigma_1}\\
d^{c1} &= \left(\frac{\xi_{15}+i\xi_{16}}{\sqrt{2}}-\frac{\xi_{21}+i\xi_{22}}{\sqrt{6}}-\frac{\xi_{27}+i\xi_{28}}{2\sqrt{3}}+\frac{\xi_{29}-i\xi_{30}}{2\sqrt{5}}+\frac{\xi_{31}-i\xi_{32}}{\sqrt{30}}\right)e^{i\sigma_1}\\
c^{c1} &= \frac{\xi_{11}-i\xi_{12}}{\sqrt{2}}e^{i\sigma_1}\\
c^{c3} &= \frac{\xi_{19}+i\xi_{20}}{\sqrt{2}}e^{i\sigma_1}\\
s^{c2} &= \left(\frac{\xi_{21}+i\xi_{22}}{\sqrt{\frac{3}{2}}}-\frac{\xi_{27}+i\xi_{28}}{2\sqrt{3}}+\frac{\xi_{29}-i\xi_{30}}{2\sqrt{5}}+\frac{\xi_{31}-i\xi_{32}}{\sqrt{30}}\right)e^{i\sigma_1}\\
t^{c1} &= \frac{\xi_{13}-i\xi_{14}}{\sqrt{2}}e^{i\sigma_1}\\
t^{c2} &= \frac{\xi_{19}-i\xi_{20}}{\sqrt{2}}e^{i\sigma_1}\\
b^{c3} &= \left(\frac{\xi_{27}+i\xi_{28}}{2\sqrt{\frac{1}{3}}}+\frac{\xi_{29}-i\xi_{30}}{2\sqrt{5}}+\frac{\xi_{31}-i\xi_{32}}{\sqrt{30}}\right)e^{i\sigma_1}\\
\nu_e &= \left(\frac{\xi_{29}+i\xi_{30}}{2\sqrt{5}}-\frac{\xi_{31}+i\xi_{32}}{\sqrt{30}}\right)e^{i(\sigma_1+\sigma_2+\sigma_3)}\\
\nu_\mu &= \left(\frac{\xi_{31}+i\xi_{32}}{\sqrt{\frac{6}{5}}}\right)e^{i(\sigma_1-\sigma_2+\sigma_3)}\\
\nu_\tau &= \left(\frac{\xi_{15}-i\xi_{16}}{\sqrt{2}}+\frac{\xi_{21}-i\xi_{22}}{\sqrt{6}}+\frac{\xi_{27}-i\xi_{28}}{2\sqrt{3}}-\frac{\xi_{29}+i\xi_{30}}{2\sqrt{5}}-\frac{\xi_{31}+i\xi_{32}}{\sqrt{30}}\right)e^{i(\sigma_1-2\sigma_3)}.
\end{aligned}
\tag{43}
$$

The non-zero elements of the U-matrix are

$$\begin{aligned}
U_{4,1} &= U_{5,1} = U_{6,1} = U_{10,1} = \frac{\sigma_1'}{\sqrt{7}} \\
U_{7,1} &= = \frac{\sigma_1' + \sigma_2' + \sigma_3'}{\sqrt{7}} \\
U_{7,2} &= = \frac{\sigma_1' + \sigma_2' + \sigma_3'}{\sqrt{2}} \\
U_{7,3} &= = \frac{\sigma_1' + \sigma_2' + \sigma_3'}{\sqrt{6}} \\
U_{8,1} &= \frac{\sigma_1' - \sigma_2' + \sigma_3'}{\sqrt{7}} \\
U_{8,2} &= \frac{\sigma_1' - \sigma_2' + \sigma_3'}{\sqrt{2}} \\
U_{8,3} &= \frac{\sigma_1' - \sigma_2' + \sigma_3'}{\sqrt{6}} \\
U_{9,1} &= \frac{\sigma_1' - 2\sigma_3'}{\sqrt{7}} \\
U_{3,9} &= \frac{\sigma_1' - 2\sigma_3'}{\sqrt{\frac{3}{2}}} \\
U_{15,16} &= \sigma_3' \\
U_{20,15} &= U_{16,19} = \frac{\sigma_1' - \sigma_3'}{\sqrt{3}} \\
U_{22,15} &= U_{16,21} = \frac{\sigma_1' - \sigma_3'}{\sqrt{6}} \\
U_{24,15} &= U_{23,16} = \frac{\sigma_1' - \sigma_3'}{\sqrt{10}} \\
U_{26,15} &= U_{25,16} = \frac{\sigma_1' - \sigma_3'}{\sqrt{15}} \\
U_{19,20} &= \frac{2\sigma_1' + \sigma_3'}{3} \\
U_{22,19} &= U_{20,21} = \frac{\sigma_1' - \sigma_3'}{3\sqrt{2}} \\
U_{24,19} &= U_{23,20} = \frac{\sigma_1' - \sigma_3'}{\sqrt{30}} \\
U_{26,19} &= U_{25,20} = \frac{\sigma_1' - \sigma_3'}{3\sqrt{5}} \\
U_{21,22} &= \frac{5\sigma_1' + \sigma_3'}{6} \\
U_{24,21} &= U_{23,22} = \frac{\sigma_1' - \sigma_3'}{2\sqrt{15}} \\
U_{26,21} &= U_{25,22} = \frac{\sigma_1' - \sigma_3'}{3\sqrt{10}} \\
U_{23,24} &= \frac{7\sigma_1' + 8\sigma_2' + 7\sigma_3'}{10} \\
U_{26,23} &= U_{24,25} = \frac{3\sigma_1' + 2\sigma_2' + 3\sigma_3'}{5\sqrt{6}} \\
U_{25,26} &= \frac{4\sigma_1' - 4\sigma_2' + 4\sigma_3'}{5}
\end{aligned} \tag{44}$$

and their antisymmetric counterparts.

The mass matrix for the physical excitations appears as (in units of φ^2)

$$
\begin{aligned}
M_{4,4} &= M_{5,5} = M_{6,6} = \frac{g_1^2}{9} + g_2^2 + \frac{4g_3^2}{3} \\
M_{4,5} &= M_{4,6} = M_{5,6} = \frac{g_1^2}{9} + g_2^2 - \frac{2g_3^2}{3} \\
M_{4,7} &= M_{4,8} = M_{4,9} = M_{5,7} = M_{5,8} = M_{5,9} = M_{6,7} = M_{6,8} = M_{6,9} = \frac{-g_1^2}{3} - g_2^2 \\
M_{4,10} &= M_{5,10} = \frac{2g_1^2}{3} \\
M_{7,7} &= M_{7,8} = M_{7,9} = M_{8,8} = M_{8,9} = M_{9,9} = g_1^2 + g_2^2 \\
M_{7,10} &= = M_{8,10} = M_{9,10} = -2g_1^2 \\
M_{10,10} &= 4g_1^2 \\
M_{11,11} &= M_{12,12} = M_{13,13} = M_{14,14} = M_{17,17} = M_{18,18} = 2g_3^2 \\
M_{15,15} &= M_{16,16} = 2g_2^2 \\
M_{15,19} &= M_{16,20} = \frac{2g_2^2}{\sqrt{3}} \\
M_{15,21} &= M_{16,22} = \frac{\sqrt{2}g_2^2}{\sqrt{3}} \\
M_{15,23} &= -M_{16,24} = \frac{3\sqrt{2}g_2^2}{\sqrt{5}} \\
M_{15,25} &= -M_{16,26} = \frac{2\sqrt{3}g_2^2}{\sqrt{5}} \\
M_{19,19} &= M_{20,20} = \frac{2g_2^2}{3} \\
M_{19,21} &= M_{20,22} = \frac{\sqrt{2}g_2^2}{3} \\
M_{19,23} &= -M_{20,24} = \frac{\sqrt{6}g_2^2}{\sqrt{5}} \\
M_{19,25} &= -M_{20,26} = \frac{2g_2^2}{\sqrt{5}} \\
M_{21,21} &= M_{22,22} = \frac{g_2^2}{3} \\
M_{21,23} &= -M_{22,24} = \frac{\sqrt{3}g_2^2}{\sqrt{5}} \\
M_{21,25} &= -M_{22,26} = \frac{\sqrt{2}g_2^2}{\sqrt{5}} \\
M_{23,23} &= M_{24,24} = \frac{9g_2^2}{5} \\
M_{23,25} &= M_{24,26} = \frac{3\sqrt{6}g_2^2}{5} \\
M_{25,25} &= M_{26,26} = \frac{6g_2^2}{5}
\end{aligned}
\tag{45}
$$

and their symmetric counterparts. The eigenvalues are $M_1^2 = \frac{11g_1^2+9g_2^2+\sqrt{121g_1^4-54g_1^2g_2^2+81g_2^4}}{3}\varphi^2$, $M_2^2 = \frac{11g_1^2+9g_2^2-\sqrt{121g_1^4-54g_1^2g_2^2+81g_2^4}}{3}\varphi^2$,$M_3^2 = M_4^2 = 6g_2^2\varphi^2$, $M_5^2 = M_6^2 = M_7^2 = M_8^2 = M_9^2 = M_{10}^2 = M_{11}^2 = M_{12}^2 = 2g_3^2\varphi^2$,$M_{13}^2 = ... = M_{26}^2 = 0$.

The J-matrix is (really: The J-matrix can be the splitting of eigenspaces of higher dimensions into subspaces is

arbitrary)

$$
\begin{aligned}
J_{4,13} &= -J_{3,14} = \frac{-\sqrt{k}+\sqrt{k+\frac{6g_2^2\varphi^2}{k}}}{4\sqrt{2}(k^2+6g_2^2\varphi^2)^{\frac{1}{4}}}\,(\sigma_2'-2\sigma_3') \\
J_{3,16} &= -J_{4,15} = \frac{-\sqrt{k}+\sqrt{k+\frac{6g_2^2\varphi^2}{k}}}{4\sqrt{30}(k^2+6g_2^2\varphi^2)^{\frac{1}{4}}}\,(5\sigma_2'+6\sigma_3') \\
J_{4,17} &= J_{3,18} = -\frac{\sqrt{3}(-\sqrt{k}+\sqrt{k+\frac{6g_2^2\varphi^2}{k}})}{4\sqrt{10}(k^2+6g_2^2\varphi^2)^{\frac{1}{4}}}\sigma_3' \\
J_{4,19} &= J_{3,20} = -\frac{-\sqrt{k}+\sqrt{k+\frac{6g_2^2\varphi^2}{k}}}{4\sqrt{2}(k^2+6g_2^2\varphi^2)^{\frac{1}{4}}}\sigma_3' \\
J_{1,24} &= \frac{\left(g_1^2+9g_2^2+\sqrt{121g_1^4-54g_1^2g_2^2+81g_2^4}\right)\left(-3\sqrt{k}+\sqrt{\frac{9k^2+3(11g_1^2+9g_2^2+\sqrt{121g_1^4-54g_1^2g_2^2+81g_2^4})\varphi^2}{k}}\right)}{3^{\frac{3}{4}}8\sqrt{14}g_1^2\sqrt{\frac{\sqrt{121g_1^4-54g_1^2g_2^2+81g_2^4}}{3g_1^2-9g_2^2+\sqrt{121g_1^4-54g_1^2g_2^2+81g_2^4}}}\left(3k^2+\left(11g_1^2+9g_2^2+\sqrt{121g_1^4-54g_1^2g_2^2+81g_2^4}\right)\varphi^2\right)^{\frac{1}{4}}}\sigma_3' \qquad (46) \\
J_{1,25} &= \frac{\left(g_1^2+9g_2^2+\sqrt{121g_1^4-54g_1^2g_2^2+81g_2^4}\right)\left(-3\sqrt{k}+\sqrt{\frac{3k^2+3(11g_1^2+9g_2^2+\sqrt{121g_1^4-54g_1^2g_2^2+81g_2^4})\varphi^2}{k}}\right)}{3^{\frac{3}{4}}8\sqrt{14}g_1^2\sqrt{\frac{\sqrt{121g_1^4-54g_1^2g_2^2+81g_2^4}}{3g_1^2-9g_2^2+\sqrt{121g_1^4-54g_1^2g_2^2+81g_2^4}}}\left(3k^2+\left(11g_1^2+9g_2^2+\sqrt{121g_1^4-54g_1^2g_2^2+81g_2^4}\right)\varphi^2\right)^{\frac{1}{4}}}\sigma_2' \\
J_{2,24} &= \frac{\left(g_1^2+9g_2^2-\sqrt{121g_1^4-54g_1^2g_2^2+81g_2^4}\right)\left(-3\sqrt{k}+\sqrt{\frac{9k^2+3(11g_1^2+9g_2^2-\sqrt{121g_1^4-54g_1^2g_2^2+81g_2^4})\varphi^2}{k}}\right)}{3^{\frac{3}{4}}8\sqrt{14}g_1^2\sqrt{\frac{\sqrt{121g_1^4-54g_1^2g_2^2+81g_2^4}}{-3g_1^2+9g_2^2+\sqrt{121g_1^4-54g_1^2g_2^2+81g_2^4}}}\left(3k^2+\left(11g_1^2+9g_2^2-\sqrt{121g_1^4-54g_1^2g_2^2+81g_2^4}\right)\varphi^2\right)^{\frac{1}{4}}}\sigma_3' \\
J_{2,25} &= \frac{\left(g_1^2+9g_2^2-\sqrt{121g_1^4-54g_1^2g_2^2+81g_2^4}\right)\left(-3\sqrt{k}+\sqrt{\frac{3k^2+3(11g_1^2+9g_2^2-\sqrt{121g_1^4-54g_1^2g_2^2+81g_2^4})\varphi^2}{k}}\right)}{3^{\frac{3}{4}}8\sqrt{14}g_1^2\sqrt{\frac{\sqrt{121g_1^4-54g_1^2g_2^2+81g_2^4}}{-3g_1^2+9g_2^2+\sqrt{121g_1^4-54g_1^2g_2^2+81g_2^4}}}\left(3k^2+\left(11g_1^2+9g_2^2-\sqrt{121g_1^4-54g_1^2g_2^2+81g_2^4}\right)\varphi^2\right)^{\frac{1}{4}}}\sigma_2'
\end{aligned}
$$

and their symmetric counterparts.

Here the J matrix show rotation between states 1-4 and the light states, giving particle production and possible preheating. The reason that the SU(3) states do not rotate is that the 3 Q's have the same SU(2)-charge, and the 2 diagonal SU(3) generators have removed the phases between them.

In fact, changing the assignments such that the quarks have split SU(2)-charges will change something, even the eigenvalues, but it will not change that J is nonzero and preheating is possible.

VII. ONE FLAT DIRECTION - SUMMARY

For the 2 flat directions mentioned as the most obvious candidate to be the inflaton in [6], QLD^c and LLE^c, we find no preheating. The reason [12] found differently with a toy model direction of 3 superfields was that it was rather special to have 3 VEV-fields and only 1 broken generator. When only 1 generator was broken, only one phase difference was removed, and the second phase difference gave the preheating. However, for LLE^c and $U^cD^cD^c$ 2 diagonal generators are broken and there is no preheating due to the diagonal generators. We think, inspired by [9], it makes sense to split the involved fields in those connected to VEV's by the diagonal generators (from here: Sector 1), and those connected to the VEV by the off-diagonal generators (from here: Sector 2). In this case, and we suspect in most others, the structure of Sector 2, is that the massive states are Higgses, and they all have the same eigenvalue. Therefore rotation does not have an effect (in fact, rotation does not make sense, since one cannot distinguish the eigenstates). In QLD^c though, they have different eigenvalues - some fields connected to the VEV through $SU(2)$, others through $SU(3)_c$. However, for each field it is either or. Any difference from this, should be if a field is connected to 2 VEV's, one by a $SU(2)$ and one by a $SU(3)_c$ generator. For Sector 1, it would take more than 3 fields (or less than 2 broken generators). This is what happens in $QLQLQLE^c$. We can gauge away 4 phase differences, but this leaves 2 phase differences that can give the preheating. There could also be a mixing between sectors, if a field was connected to 1 VEV by a diagonal generator and to another by an off-diagonal one. However, this seems impossible for a single flat direction.

VIII. $U^cD^cD^c$, LLE^c SIMULTANIOUSLY

The 2 directions first presented can co-exist. In fact, there is no reason why they should not both get large VEV's [9]. It is not so easy to argue why there should be no preheating - since now we have 6 Sector 1 fields, and only 4 diagonal generators to break.

$$\begin{aligned}
< u^{c\bar{1}} >&= \varphi e^{i\sigma_1} \\
< s^{c\bar{2}} >&= \varphi e^{i\sigma_2} \\
< b^{c\bar{3}} >&= \varphi e^{i\sigma_3} \\
< \nu_e >&= A\varphi e^{i\sigma_4} \\
< \mu >&= A\varphi e^{i\sigma_5} \\
< \tau^c >&= A\varphi e^{i\sigma_6}
\end{aligned} \tag{47}$$

where A is the relation between the absolute value of the VEV's involved. The Lagrangian reads

$$\mathcal{L} = \sum_{i=1}^{3} \frac{1}{2} |D_\mu \Phi_i|^2 - V - \frac{1}{4} F_{\mu\nu}^2 - \frac{1}{4} \sum_i W_{\mu\nu}^{i2} - \frac{1}{4} \sum_i G_{\mu\nu}^{i2} \tag{48}$$

where for field ϕ_i $D_i^\mu = \left[(\partial^\mu - iq_i A_0^\mu)\delta_{ij} - \sum_{a=1}^{3} iP_{ij}^a A_a^\mu - \sum_{A=1}^{8} iGM_{ij}^A B_A^\mu \right] \phi_j$ denotes the covariant derivative. The potential now looks like

$$V = \frac{1}{2} \left(D_H^2 + \sum_a D_a^2 + \sum_A D_A^2 \right). \tag{49}$$

To remove mixed kinetic terms we must reparametrise

$$\begin{aligned}
u^{c\bar{1}} &= (\varphi + \xi_2) e^{i(\sigma_1 + \frac{\xi_1}{\sqrt{3}\varphi})}, \\
u^{c\bar{2}} &= \frac{\xi_5 + i\xi_6}{\sqrt{2}} e^{i\sigma_1}, \\
u^{c\bar{3}} &= \frac{\xi_7 + i\xi_8}{\sqrt{2}} e^{i\sigma_1}, \\
s^{c\bar{1}} &= \frac{\xi_5 - i\xi_6}{\sqrt{2}} e^{i\sigma_1}, \\
s^{c\bar{2}} &= (\varphi + \xi_3) e^{i(\sigma_1 + \frac{\xi_1}{\sqrt{3}\varphi})} \\
s^{c\bar{3}} &= \frac{\xi_9 + i\xi_{10}}{\sqrt{2}} e^{i\sigma_1}, \\
b^{c\bar{1}} &= \frac{\xi_7 - i\xi_8}{\sqrt{2}} e^{i\sigma_1}, \\
b^{c\bar{2}} &= \frac{\xi_9 - i\xi_{10}}{\sqrt{2}} e^{i\sigma_1}, \\
b^{c\bar{3}} &= (\varphi + \xi_4) e^{i(\sigma_1 + \frac{\xi_1}{\sqrt{3}\varphi})} \\
\nu_e &= (A\varphi + \xi_{12}) e^{i(\sigma_2 + \frac{\xi_{11}}{\sqrt{3}A\varphi})}, \\
e &= \frac{\xi_{15} + i\xi_{16}}{\sqrt{2}} e^{i\sigma_2}, \\
\nu_\mu &= \frac{\xi_{15} - i\xi_{16}}{\sqrt{2}} e^{i\sigma_2}, \\
\mu &= (A\varphi + \xi_{13}) e^{i(\sigma_2 + \frac{\xi_{11}}{\sqrt{3}A\varphi})} \\
\tau^c &= (A\varphi + \xi_{14}) e^{i(\sigma_2 + \frac{\xi_{11}}{\sqrt{3}A\varphi})}
\end{aligned} \tag{50}$$

After verifying that the mixed derivatives have indeed been removed, we use coordinate derivatives, and find the

remaining kinetic terms (those not to second order) to be: (to zero'th order in φ)

$$
\begin{aligned}
\mathcal{L} \supset\; & \sum_{i=1}^{16}\left(\frac{1}{2}\dot{\xi_i}^2\right)+\sum_{i=2}^{10}\left(\frac{1}{2}\xi_i^2\dot{\sigma_1}^2\right)+\sum_{i=12}^{16}\left(\frac{1}{2}\xi_i^2\dot{\sigma_2}^2\right) \\
& +\sum_{i=2}^{4}\left(\varphi\xi_i\dot{\sigma_1}^2+\frac{2}{\sqrt{3}}\xi_i\dot{\xi_1}\right)+\sum_{i=12}^{14}\left(A\varphi\xi_i\dot{\sigma_2}^2+\frac{2}{\sqrt{3}}\xi_i\dot{\xi}_{11}\right) \\
& +\frac{3}{2}\varphi^2\dot{\sigma_1}^2+\frac{3}{2}A^2\varphi^2\dot{\sigma_2}^2+\sqrt{3}\varphi\dot{\sigma_1}\dot{\xi_1}+\sqrt{3}A\varphi\dot{\sigma_2}\dot{\xi_{11}}.
\end{aligned}
\tag{51}
$$

It seems the 2 directions do not "see" each other. We find (after antisymmetrising)

$$
U=\begin{pmatrix}
0 & -\frac{\dot{\sigma_1}}{\sqrt{3}} & -\frac{\dot{\sigma_1}}{\sqrt{3}} & -\frac{\dot{\sigma_1}}{\sqrt{3}} & 0&0&0&0&0&0&0&0&0&0&0&0\\
\frac{\dot{\sigma_1}}{\sqrt{3}} & 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
\frac{\dot{\sigma_1}}{\sqrt{3}} & 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
\frac{\dot{\sigma_1}}{\sqrt{3}} & 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0& -\frac{\dot{\sigma_2}}{\sqrt{3}} & -\frac{\dot{\sigma_2}}{\sqrt{3}} & -\frac{\dot{\sigma_2}}{\sqrt{3}} &0&0\\
0&0&0&0&0&0&0&0&0&0&\frac{\dot{\sigma_2}}{\sqrt{3}}&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&\frac{\dot{\sigma_2}}{\sqrt{3}}&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&\frac{\dot{\sigma_2}}{\sqrt{3}}&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0
\end{pmatrix}
$$

This looks as if the 2 parts are completely separated. The mass matrix for the physical excitations appears as

$$
\mathcal{M}^2=\varphi^2 \tag{52}
$$

$$
\begin{pmatrix}
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&\frac{16g_1^2+12g_3^2}{9}&\frac{-8g_1^2-6g_3^2}{9}&\frac{8g_1^2+6g_3^2}{9}&0&0&0&0&0&0&0&\frac{4Ag_1^2}{3}&\frac{4Ag_1^2}{3}&\frac{-8Ag_1^2}{3}&0&0\\
0&\frac{-8g_1^2-6g_3^2}{9}&\frac{4g_1^2+12g_3^2}{9}&\frac{4g_1^2-6g_3^2}{9}&0&0&0&0&0&0&0&\frac{-2Ag_1^2}{3}&\frac{-2Ag_1^2}{3}&\frac{4Ag_1^2}{3}&0&0\\
0&\frac{-8g_1^2-6g_3^2}{9}&\frac{4g_1^2-6g_3^2}{9}&\frac{4g_1^2+12g_3^2}{9}&0&0&0&0&0&0&0&\frac{-2Ag_1^2}{3}&\frac{-2Ag_1^2}{3}&\frac{4Ag_1^2}{3}&0&0\\
0&0&0&0&2g_3^2&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&2g_3^2&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&2g_3^2&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&2g_3^2&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&2g_3^2&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&2g_3^2&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&\frac{4Ag_1^2}{3}&\frac{-2Ag_1^2}{3}&\frac{-2Ag_1^2}{3}&0&0&0&0&0&0&0&A^2g_1^2+A^2g_2^2&A^2g_1^2-A^2g_2^2&-2A^2g_1^2&0&0\\
0&\frac{4Ag_1^2}{3}&\frac{-2Ag_1^2}{3}&\frac{-2Ag_1^2}{3}&0&0&0&0&0&0&0&A^2g_1^2-A^2g_2^2&A^2g_1^2+A^2g_2^2&-2A^2g_1^2&0&0\\
0&\frac{-8Ag_1^2}{3}&\frac{4Ag_1^2}{3}&\frac{4Ag_1^2}{3}&0&0&0&0&0&0&0&-2A^2g_1^2&-2A^2g_1^2&4A^2g_1^2&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&2A^2g_2^2&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&2A^2g_3^2
\end{pmatrix}
\tag{53}
$$

with eigenvalues

$$
\begin{aligned}
M_1^2 &= \left(\frac{4+9A^2}{3}g_1^2+g_3^2+\frac{1}{3}\sqrt{(4+9A^2)^2g_1^4-6(-4+9A^2)g_1^2g_3^2+9g_3^4}\right)\varphi^2 \\
M_2^2 &= \left(\frac{4+9A^2}{3}g_1^2+g_3^2-\frac{1}{3}\sqrt{(4+9A^2)^2g_1^4-6(-4+9A^2)g_1^2g_3^2+9g_3^4}\right)\varphi^2 \\
M_3^2 &= M_4^2=M_5^2=M_6^2=M_7^2=M_8^2=M_9^2=2g_3^2\varphi^2 \\
M_{10}^2 &= M_{11}^2=M_{12}^2=2A^2g_2^2\varphi^2 \\
M_{13}^2 &= M_{14}^2=M_{15}^2=M_{16}^2=0.
\end{aligned}
\tag{54}
$$

Even though this indeed looks like a mixing between the two directions, again $J=0$ (16 by 16 matrix) and there is no particle production and therefore no preheating.

IX. $QLD^c + LLE^c$ - AN OVERLAPPING DIRECTION

While $LLE^c + QLD^c$ shows that more than 1 phase is not enough to secure preheating, it is clear that the 2 directions did not overlap. There are flat directions more intimately connected. One example of this is $QLD^c + LLE^c$ - with one L-field in common. This is exiting, since here the flat directions cannot just have one phase each. Q and D^c must be from different generations and have the same (or opposite, if you like) colour charge. It is easy to show that flatness is independent of phase, and that the common field shall have a VEV that is the square root of the sum of squares of the VEV's from the 2 directions.

$$\begin{aligned}
< d^{c^1} >&= A\varphi e^{i\sigma_4} \\
< s^{c\bar{1}} >&= A\varphi e^{i\sigma_5} \\
< \nu_e >&= \sqrt{1+A^2}\varphi e^{i\sigma_3} \\
< \mu >&= \varphi e^{i\sigma_2} \\
< \tau^c >&= \varphi e^{i\sigma_1}
\end{aligned} \tag{55}$$

where A is the relation between the absolute values of the VEV's. The Lagrangian, covariant derivatives and the potential looks as before.

To remove mixed kinetic terms, we must re-parameterise

$$\begin{aligned}
u^{c^1} &= \left(\frac{\xi_8 + i\xi_9}{\sqrt{1+A^2}} + \frac{A(\xi_{14} - i\xi_{15})}{\sqrt{2(1+A^2)}}\right) e^{i\left(\sigma + \left(-1 - \frac{1-A^2}{5(1+A^2)}\right)\gamma\right)} \\
d^{c^1} &= (A\varphi + \xi_4) e^{i\left((\sigma + \frac{1}{\sqrt{5}}\xi_6) + \left(-1 - \frac{1-A^2}{5(1+A^2)}\right)\left(\gamma + \frac{\sqrt{5}(1+A^2)}{2\sqrt{6+8A^2+6A^4}}\xi_7\right)\right)} \\
d^{c^2} &= \frac{\xi_{10} + i\xi_{11}}{\sqrt{2}} e^{i\left(\sigma + \left(-1 - \frac{1-A^2}{5(1+A^2)}\right)\gamma\right)} \\
d^{c^3} &= \frac{\xi_{12} + i\xi_{13}}{\sqrt{2}} e^{i\left(\sigma + \left(-1 - \frac{1-A^2}{5(1+A^2)}\right)\gamma\right)} \\
s^{c\bar{1}} &= (A\varphi + \xi_5) e^{i\left((\sigma + \frac{1}{\sqrt{5}}\xi_6) + \left(-1 - \frac{1-A^2}{5(1+A^2)}\right)\left(\gamma + \frac{\sqrt{5}(1+A^2)}{2\sqrt{6+8A^2+6A^4}}\xi_7\right)\right)} \\
s^{c\bar{2}} &= \frac{-\xi_{10} + i\xi_{11}}{\sqrt{2}} e^{i\left(\sigma + \left(-1 - \frac{1-A^2}{5(1+A^2)}\right)\gamma\right)} \\
s^{c\bar{3}} &= \frac{-\xi_{12} + i\xi_{13}}{\sqrt{2}} e^{i\left(\sigma + \left(-1 - \frac{1-A^2}{5(1+A^2)}\right)\gamma\right)} \\
\nu_e &= (\sqrt{1+A^2}\varphi + \xi_3) e^{i\left((\sigma + \frac{1}{\sqrt{5}}\xi_6) + \frac{4(1-A^2)}{5(1+A^2)}\left(\gamma + \frac{\sqrt{5}(1+A^2)}{2\sqrt{6+8A^2+6A^4}}\xi_7\right)\right)} \\
e &= \frac{\xi_{14} + i\xi_{15}}{\sqrt{2}} e^{i\left(\sigma + \frac{4(1-A^2)}{5(1+A^2)}\gamma\right)} \\
\nu_\mu &= \left(\frac{-A(\xi_8 + i\xi_9)}{\sqrt{1+A^2}} + \frac{\xi_{14} - i\xi_{15}}{\sqrt{2(1+A^2)}}\right) e^{i\left(\sigma + \left(1 - \frac{1-A^2}{5(1+A^2)}\right)\gamma\right)} \\
\mu &= (\varphi + \xi_2) e^{i\left((\sigma + \frac{1}{\sqrt{5}}\xi_6) + \left(1 - \frac{1-A^2}{5(1+A^2)}\right)\left(\gamma + \frac{\sqrt{5}(1+A^2)}{2\sqrt{6+8A^2+6A^4}}\xi_7\right)\right)} \\
\tau^c &= (\varphi + \xi_1) e^{i\left((\sigma + \frac{1}{\sqrt{5}}\xi_6) + \left(1 - \frac{1-A^2}{5(1+A^2)}\right)\left(\gamma + \frac{\sqrt{5}(1+A^2)}{2\sqrt{6+8A^2+6A^4}}\xi_7\right)\right)}
\end{aligned} \tag{56}$$

There are quite many kinetic terms now, but they include

$$\mathcal{L} \supset \sum_{i=1}^{15} \left(\frac{1}{2}\dot{\xi_i}^2\right) \tag{57}$$

and no cross terms. The U-matrix is (after antisymmetrising)

$$\begin{aligned}
U_{1,6} &= -U_{6,1} = U_{2,6} = -U_{6,2} = \frac{(4+6A^2)\dot{\gamma} + 5(1+A^2)\dot{\sigma}}{5\sqrt{5}(1+A^2)} \\
U_{1,7} &= -U_{7,1} = U_{2,7} = -U_{7,2} = \frac{(2+3A^2)((4+6A^2)\dot{\gamma} + 5(1+A^2)\dot{\sigma})}{5\sqrt{10}(1+A^2)\sqrt{3+4A^2+3A^4}} \\
U_{3,6} &= -U_{6,3} = \frac{4(1-A^2)\dot{\gamma} + 5(1+A^2)\dot{\sigma}}{5\sqrt{5}(1+A^2)} \\
U_{3,7} &= -U_{7,3} = \frac{\sqrt{2}(-1+A^2)(4(-1+A^2)\dot{\gamma} - 5(1+A^2)\dot{\sigma})}{5\sqrt{5}(1+A^2)\sqrt{3+4A^2+3A^4}} \\
U_{4,6} &= -U_{6,4} = U_{5,6} = -U_{6,5} = \frac{-(6+4A^2)\dot{\gamma} + 5(1+A^2)\dot{\sigma}}{5\sqrt{5}(1+A^2)} \\
U_{4,7} &= -U_{7,4} = U_{5,7} = -U_{7,5} = \frac{(3+2A^2)((6+4A^2)\dot{\gamma} - 5(1+A^2)\dot{\sigma})}{5\sqrt{10}(1+A^2)\sqrt{3+4A^2+3A^4}} \\
U_{8,9} &= -U_{9,8} = \frac{6(-1+A^2)\dot{\gamma}}{5(1+A^2)} + \dot{\sigma} \\
U_{8,15} &= -U_{15,8} = U_{9,14} = -U_{14,9} = \frac{\sqrt{2}A\dot{\gamma}}{(1+A^2)} \\
U_{remaining} &= 0.
\end{aligned} \tag{58}$$

This looks as if everything mixes - certainly it does not look as if there are 2 separable parts. The mass matrix is quite complicated, but the structure is like this

$$\mathcal{M}^2 = \varphi^2 \begin{pmatrix} M_{7\times 7} & 0_{7\times 8} \\ 0_{8\times 7} & D_{8\times 8} \end{pmatrix} \tag{59}$$

where M is the mass matrix (with no 0-entries) for the previously mentioned sector 1, while D is a diagonal matrix which is for the previously mentioned sector 2. So, the sectors are clearly separated. The sector 1 part, has 3 very complicated eigenvalues, and zero is eigenvalue with multiplicity of 4. The sector 2 part has eigenvalues (entries) ordered after the ξ-fields: $(0, 0, 2A^2g_3^2, 2A^2g_3^2, 2A^2g_3^2, 2A^2g_3^2, 2(1+A^2)g_2^2, 2(1+A^2)g_2^2)\varphi^2$. Also, it is important to notice that M is time-dependant. All the elements involving 6 or 7 are time dependent.

The elements (symmetry implied) are

$$
\begin{aligned}
M_{1,1} &= 4g_1^2, M_{1,2} = -2g_1^2, M_{1,3} = -2\sqrt{1+A^2}g_1^2, M_{1,4} = \frac{2Ag_1^2}{3}, M_{1,5} = \frac{4Ag_1^2}{3} \\
M_{2,2} &= g_1^2 + g_2^2, M_{2,3} = \sqrt{1+A^2}(g_1^2 - g_2^2), M_{2,4} = A((-g_1^2/3) + g_2^2), M_{2,5} = \frac{-2Ag_1^2}{3} \\
M_{3,3} &= (1+A^2)(g_1^2 + g_2^2), M_{3,4} = -\frac{1}{3}A\sqrt{1+A^2}(g_1^2 + 3g_2^2), M_{3,5} = -\frac{2}{3}A\sqrt{1+A^2}g_1^2 \\
M_{4,4} &= \frac{A^2(g_1^2 + 9g_2^2 + 12g_3^2)}{9}, M_{4,5} = \frac{2(g_1^2 - 6g_3^2)A^2}{9}, M_{5,5} = \frac{4A^2(g_1^2 + 3g_3^2)}{9} \\
M_{1,6} &= \frac{2g_1^2\left((1+A^2)\sin[\frac{(8(-1+A^2)\gamma)}{5(1+A^2)} - 2\sigma] + \sqrt{1+A^2}\left(-A\sin[\frac{4((3+2A^2)\gamma)}{5(1+A^2)} - 2\sigma] + \sin[\frac{4(2+3A^2)\gamma}{5(1+A^2)} + 2\sigma]\right)\right)}{\sqrt{5}\sqrt{1+A^2}} \\
M_{1,7} &= \frac{\sqrt{\frac{2}{5}}g_1^2\left(2(1-A^4)\sin[\frac{8(-1+A^2)\gamma}{5(1+A^2)} - 2\sigma] + \sqrt{1+A^2}\left(A(3+2A^2)\sin[\frac{4(3+2A^2)\gamma}{5(1+A^2)} - 2\sigma] + (2+3A^2)\sin[\frac{4(2+3A^2)\gamma}{5(1+A^2)} + 2\sigma]\right)\right)}{\sqrt{3+7A^2+7A^4+3A^6}} \\
M_{2,6} &= -\frac{(g_1^2 - g_2^2)\left((1+A^2)\sin[\frac{8(-1+A^2)\gamma}{5(1+A^2)} - 2\sigma] + \sqrt{1+A^2}\left(-A\sin[\frac{4(3+2A^2)\gamma}{5(1+A^2)} - 2\sigma] + \sin[\frac{4(2+3A^2)\gamma}{5(1+A^2)} + 2\sigma]\right)\right)}{\sqrt{5}\sqrt{1+A^2}} \\
M_{2,7} &= \frac{(g_1^2 - g_2^2)\left(2(A^4-1)\sin[\frac{8(-1+A^2)\gamma}{5(1+A^2)} - 2\sigma] - \sqrt{1+A^2}\left(A(3+2A^2)\sin[\frac{4(3+2A^2)\gamma}{5(1+A^2)} - 2\sigma] + (2+3A^2)\sin[\frac{4(2+3A^2)\gamma}{5(1+A^2)} + 2\sigma]\right)\right)}{\sqrt{10}\sqrt{3+7A^2+7A^4+3A^6}} \\
M_{3,6} &= -\frac{(g_1^2 + g_2^2)\left((1+A^2)\sin[\frac{8(-1+A^2)\gamma}{5(1+A^2)} - 2\sigma] + \sqrt{1+A^2}((-A\sin[\frac{4((3+2A^2))\gamma}{5(1+A^2)} - 2\sigma] + \sin[\frac{4(2+3A^2)\gamma}{5(1+A^2)} + 2\sigma]))\right)}{\sqrt{5}} \\
M_{3,7} &= -\frac{(g_1^2 + g_2^2)\left(2(1-A^4)\sin[\frac{8(-1+A^2)\gamma}{5(1+A^2)} - 2\sigma] + \sqrt{1+A^2}\left(A(3+2A^2)\sin[\frac{4(3+2A^2)\gamma}{5(1+A^2)} - 2\sigma] + (2+3A^2)\sin[\frac{4((2+3A^2))\gamma}{5(1+A^2)} + 2\sigma)\right)\right)}{\sqrt{10}\sqrt{3+4A^2+3A^4}} \\
M_{4,6} &= \frac{A(g_1^2 + 3g_2^2)\left((1+A^2)\sin[\frac{8(-1+A^2)\gamma}{5(1+A^2)} - 2\sigma] + \sqrt{1+A^2}\left(-A\sin[\frac{4(3+2A^2)\gamma}{5(1+A^2)} - 2\sigma] + \sin[\frac{4(2+3A^2)\gamma}{5(1+A^2)} + 2\sigma]\right)\right)}{3\sqrt{5}\sqrt{1+A^2}} \\
M_{4,7} &= \frac{A(g_1^2 + 3g_2^2)\left(2(1-A^4)\sin[\frac{8(-1)+A^2)\gamma}{5(1+A^2)} - 2\sigma] + \sqrt{1+A^2}\left(A(3+2A^2)\sin[\frac{4(3+2A^2)\gamma}{5(1+A^2)} - 2\sigma] + (2+3A^2)\sin[\frac{4(2+3A^2)\gamma}{5(1+A^2)} + 2\sigma]\right)\right)}{3\sqrt{10}\sqrt{3+7A^2+7A^4+3A^6}} \\
M_{5,6} &= \frac{2Ag_1^2\left((1+A^2)\sin[\frac{8(-1+A^2)\gamma}{5(1+A^2)} - 2\sigma] + \sqrt{1+A^2}\left(-A\sin[\frac{4(3+2A^2)\gamma}{5(1+A^2)} - 2\sigma] + \sin[\frac{4(2+3A^2)\gamma}{5((1+A^2)} + 2\sigma]\right)\right)}{3\sqrt{5}\sqrt{1+A^2}} \\
M_{5,7} &= \frac{\sqrt{\frac{2}{5}}Ag_1^2\left(-2(-1+A^4)\sin[\frac{8(-1+A^2)\gamma}{5(1+A^2)} - 2\sigma] + \sqrt{1+A^2}\left(A(3+2A^2)\sin[\frac{4(3+2A^2)\gamma}{5(1+A^2)} - 2\sigma] + (2+3A^2)\sin[\frac{4(2+3A^2)\gamma}{5(1+A^2)} + 2\sigma]\right)\right)}{3\sqrt{3+7A^2+7A^4+3A^6}}
\end{aligned}
\tag{60}
$$

$M_{6,6}, M_{6,7}, M_{7,7}$ are much more complicated and are omitted here.

X. $QLD^c + LLE^c$ - PREHEATING

In this case, where the mass matrix is time dependent, we must redo eq.32 and we find

$$C^T\dot{C} = B^T A^T(\dot{A}B + A\dot{B}) = -B^T U B + B^T\dot{B} \tag{61}$$

and therefore

$$
\begin{aligned}
J &= \frac{1}{2}\left(\sqrt{\omega}\,C^T\dot{C}\frac{1}{\sqrt{\omega}} - \frac{1}{\sqrt{\omega}}\,C^T\dot{C}\sqrt{\omega}\right) \\
&= \frac{1}{2}\left(\sqrt{\omega}\,(-B^T UB)\frac{1}{\sqrt{\omega}} - \frac{1}{\sqrt{\omega}}\,(-B^T UB)\sqrt{\omega}\right) + \frac{1}{2}\left(\sqrt{\omega}\,B^T\dot{B}\frac{1}{\sqrt{\omega}} - \frac{1}{\sqrt{\omega}}\,B^T\dot{B}\sqrt{\omega}\right) \\
&= J_1 + J_2(\textbf{with the obvious definition}).
\end{aligned}
\tag{62}
$$

It has been shown numerically that J_1 has the following structure - treating everything but ξ_i, σ, γ as constants -

$$J_1 = \begin{pmatrix}
0 & 0 & 0 & NZ & NZ & NZ & NZ & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & NZ & NZ & NZ & NZ & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & NZ & NZ & NZ & NZ & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
NZ & NZ & NZ & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
NZ & NZ & NZ & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
NZ & NZ & NZ & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
NZ & NZ & NZ & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & NZ \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & NZ & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & NZ & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & NZ & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix} \tag{63}$$

where NZ stands for a nonzero element. Since the columns of B were chosen such that the first 3 columns represent the massive states of sector 1, the next 4 the massless states in sector 1, and the last 8 columns represent the states of sector 2 - in the order in which the eigenvalues were mentioned, this represents particle production from the rotation between the 3 massive states and the massless states of sector 1, and particle production from the rotation between 2 of the massive and the massless states of sector 2. Reassuringly, there is no particle producing rotations between (indistinguishable) states of the same mass. Also it is easy to see that J_2 will not alter this picture, since the last 8 columns of B are constant. This make the last 8 columns of $\dot{B}$ (and therefore of $B^T\dot{B}$) zero and since multiplying by diagonal matrices cannot chance zero entries, it is clear that at least the last 8 columns of J are identical to the last 8 columns of J_1. Therefore the J-matrix is definitely nonzero, and there is particle production from the rotating eigenstates. It was shown in [12] that in general all phases will have nontrivial dynamics, which is necessary for the conclusion that J is nonzero.

XI. THE METHOD OF FIELD COUNTING - AND ITS LIMITS

A simpler approach for determining if preheating is possible is to count the fields, establishing the number of broken generators and thus the number of Goldstones and the number of Higgses and by subtraction finding the number of remaining, physical light degrees of freedom [9]. This can be done sector by sector.

LLE^c breaks $SU(2) \times U(1)$ completely. It has 6 fields (real) in sector 1, it breaks 2 diagonal generators and thus have 2 Higgses and 2 Goldstones. This leaves 2 light degrees of freedom, which corresponds to the flat direction. (It is clear that the sum of the phases cannot be gauged away.) However, one needs to argue why the 2 Higgses cannot rotate between each other - since they have different eigenvalues a possible rotation would be physical, and why the flat direction stays out of this rotation. In this case the flat direction - understood as 2 real fields, that is the direction itself, and the field combination that is orthogonal to it in all superfields individually - stays constant. So it is clearly not rotating. Sector 2 is very easy. There are 4 fields, and 2 broken off-diagonal generators and therefore 2 Higgses and 2 Goldstones. Since the 2 Higgses have the same eigenvalue, there will surely not be particle production in this sector.

$U^cD^cD^c$ breaks $SU(3)_c \times U(1)$ to $U(1)_{NEW}$. It has 6 fields in sector 1, it breaks 2 diagonal generators and thus have 2 Higgses and 2 Goldstones. This leaves 2 light degrees of freedom, which corresponds to the flat direction. However, as before, one needs to argue why the 2 Higgses cannot rotate between each other. The flat directions stay out of this rotation. Sector 2 is again very easy. There are 12 fields, and 6 broken off-diagonal generators and therefore 6 Higgses and 6 Goldstones. Since the 2 Higgses have the same eigenvalue, there will surely not be particle production in this sector.

$QLQLQLE^c$ breaks $SU(3)_c \times SU(2) \times U(1)$ completely. It has 14 fields in sector 1, it breaks 4 diagonal generators and thus have 4 Higgses and 4 Goldstones. This leaves 6 light degrees of freedom, corresponding to the 1 flat direction and 4 additional light degrees of freedom to which there can be rotations which give preheating. In Sector 2 there are 38 fields, and 8 broken off-diagonal generators and therefore 8 Higgses and 8 Goldstones. Since each field is exclusively connected to the VEV by $SU(3)$ or $SU(2)$ the Higgses cannot rotate between each other. 12 fields are completely decoupled (those of Q, differing in both $SU(3)_c$ and $SU(2)$-charge from the VEV).Indeed there is rotation to some of the remaining 10 states, but it is hard to argue exactly why and to how many, without doing the full investigation.

$LLE^c + U^cD^cD^c$ breaks $SU(3)_c \times SU(2) \times U(1)$ completely. It has 12 fields in sector 1, it breaks 4 diagonal generators and thus have 4 Higgses and 4 Goldstones. This leaves 4 light degrees of freedom, corresponding to the 2 flat directions. Again one needs to argue why the 4 Higgses cannot rotate between each other. The flat direction clearly stays out of the rotation. In Sector 2 there are 16 fields, and 8 broken off-diagonal generators and therefore 8 Higgses and 8 Goldstones. Since each field is exclusively connected to the VEV by $SU(3)$ or $SU(2)$ the Higgses cannot rotate between each other.

$QLD^c + LLE^c$ breaks $SU(3)_c \times SU(2) \times U(1)$ to $SU(2)_c$. It has 10 fields in sector 1 (one complex field in common), it breaks 3 diagonal generators and thus have 3 Higgses and 3 Goldstones. This leaves 4 light degrees of freedom, corresponding to the 2 flat directions. Again one needs to argue why the 3 Higgses cannot rotate between each other and why in this case the light fields corresponding to the flat directions *does rotate* with the Higgses. In Sector 2 there are 18 fields and 6 broken off-diagonal generators and therefore 6 Higgses and 6 Goldstones and 4 are completely decoupled. This leaves 2 light fields that can rotate. One can also argue, that the 4 down fields in Q and 4 strange fields in D^c must represent the 4 color Higgses and 4 color Goldstones with no particle production.

It seems clear, that while this counting is a nice tool to look for opportunities for preheating and to exclude preheating especially in sector 2, it is still necessary to do the full analysis in the unitary gauge to draw firm conclusions - at least when it comes to the role of the fields corresponding to the flat directions themselves.

XII. SUMMARY AND CONCLUSION

For the conclusions on one flat direction, see section VII. For 2 flat directions, we have found that particle production is possible in $QLD^c + LLE^c$ but not in $U^cD^cD^c + LLE^c$. The difference seems to be the presence of a common field in the former case, but not in the latter. We have also found that it is necessary to transform to the unitary gauge after identifying the Goldstones, in order to make correct conclusions. Counting fields and broken generators can give hints to whether there is particle production or not, but it is not sufficient for firm conclusions.

Finally, we shall stress that what we have shown is that there will be particle production in the QLD^c+LLE^c case. However, the statement that both directions are likely to get large VEV's [9] has not been investigated in this paper. Neither has the very recent claim that even if non-perturbative particle production happen, the main decay mode will still be perturbative [11]. Also, whether the rotation of the flat directions are fast enough for this particle production to lead to preheating and thus not giving the effect of delayed thermalisation is outside the scope of the present paper. We presume that the situation is close to the situation in [12] - and that there will be very significant particle production. However, as stated in [12], the effect of SUSY breaking terms in the Lagrangian has not been taken into account.

But we can conclude that in order to determine the role of SUSY flat directions in (p)reheating, it is absolutely necessary to determine which flat directions get the large VEVs (only a limited number of the countless flat directions can get large VEVs at the same time) and if many directions get a large VEV a numerical study will probably be necessary to determine if the role of some additional flat directions can be ignored.

XIII. ACKNOWLEDGEMENTS

I would like to thank David Maybury, Francesco Riva and Stephen M West for the collaboration that led to the formalism used in the present paper. I would also like to thank Steen Hannestad and Martin S Sloth for useful discussions.

[1] T. Gherghetta, C. F. Kolda and S. P. Martin, Nucl. Phys. B **468**, 37 (1996).
[2] For a review, see K. Enqvist and A. Mazumdar, Phys. Rept. **380**, 99 (2003).
[3] I. Affleck and M. Dine, Nucl. Phys. B **249**, 361 (1985).
[4] A. D. Linde, Phys. Lett. B **160**, 243 (1985).
[5] M. Dine, L. Randall and S. D. Thomas, Phys. Rev. Lett. **75**, 398 (1995).
[6] See eg., R. Allahverdi, K. Enqvist, J. Garcia-Bellido and A. Mazumdar, Phys. Rev. Lett. **97** (2006) 191304.
[7] R. Allahverdi and A. Mazumdar, JCAP **0610**, 008 (2006). [arXiv:hep-ph/0512227v2].
[8] R. Allahverdi and A. Mazumdar, Phys. Rev. D **76**, 103526 (2007).
[9] K. A. Olive and M. Peloso, Phys. Rev. D **74**, 103514 (2006).
[10] R. Allahverdi and A. Mazumdar, J. Cosmol. Astropart. Phys. 08 (2007) 023.
[11] R. Allahverdi and A. Mazumdar, [arXiv:hep-ph/0802.4430v1].
[12] A. Basboll, D. Maybury, Francesco Riva and S. M. West, Phys. Rev. D **76**, 065005 (2007).
[13] T. W. B. Kibble Phys. Rev. **155** (1967) 1554.
[14] H. P. Nilles, M. Peloso and L. Sorbo, JHEP **0104** (2001) 004.
[15] R. Casadio, P. L. Iafelice and G. P. Vacca, Nucl. Phys. **B783**,1 (2007).